

## ABSTRACTS

### Monday

#### PLENARY LECTURE

##### DEVELOPING MATHEMATICAL CREATIVITY AND EXPERTISE IN STUDENTS AND TEACHERS: FOCUSING ON MULTIPLE SOLUTION AND INVESTIGATION TASKS

Roza Leikin  
University of Haifa, Israel

**Abstract.** *I consider developing mathematical expertise and mathematical creativity in each student as two equivalent purposes of school mathematics education. Multiple Solution Tasks and Investigation Tasks are exemplified as main tools for the identification and development of students' mathematical expertise and creativity. While development of mathematical expertise is not necessarily applies advancement of mathematical creativity, creativity-directed learning advances problem-solving expertise. I argue that development of creativity in students entails the creation of a "creative" learning environment and thus requires expertise and creativity in mathematics teachers.*

#### WORKSHOPS

##### DESIGN OF PROBLEMS FOR RESEARCH PURPOSES WITH MATHEMATICALLY TALENTED STUDENTS

Rafael Ramírez<sup>1</sup>, Juan Miguel Ribera<sup>2</sup>, María José Beltrán-Meneu<sup>3</sup>, Adela Jaime<sup>3</sup>, Ángel Gutiérrez<sup>3</sup>

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**Abstract.** *In this workshop, through specific examples, we propose the attendees to "create" their own problems so that they fit specific research needs. The process for the design is based on reviewing the appropriate bibliography, consulting databases and programs addressed to talent, selecting related problems and modifying their components. Participants in this workshop will be organized in small groups and will make their own proposals, which will be discussed in the whole group, where the proposed schema for the design of tasks related to specific research will be completed.*

## **PROBLEM SOLVING IN CONTEXT: A VARIETY OF EXTENSIONS OF PROBLEMS**

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Lyceum “Physical-Technical High School”, St.Petersburg, Russia

**Abstract.** *The idea of the workshop consists in presenting several lessons for children aged 16-18. Each lesson will start with a mathematical problem which further will be generalized in different senses. Most of these problems are taken from the book by Ivanov (2017). It is assumed that the participants will take an active part in solving the problems and discussing the related pedagogical ideas. Moreover, we hope that sometimes the participants will determine the development of the suggested theme.*

## Tuesday

### PLENARY LECTURE

#### CREATIVITY AND IMAGINATION IN MATHEMATICS

Constantinos Christou  
University of Cyprus

**Abstract.** *Much research from around the world suggests that creativity can sometimes be viewed as an unwanted distraction from the aim of the lesson (Beghetto, 2007; Kennedy, 2005). Teaching mathematics without providing for creativity denies all students the opportunity to appreciate the beauty of mathematics and fails to provide students an opportunity to fully develop the understanding of mathematical concepts. The purpose of the article is to discuss a few factors that allow for students to experience creativity and imagination in the mathematics classroom. The discussion will be structured within a framework derived from recent research. It appears that most of the researchers' contribution to the theory can be categorized into two main sources of creativity, the internal and external factors. In this article much emphasis is given on the concept of imagination as the corner stone of creativity. Thus, we describe ways in which imagination can be developed by looking at problems through different lenses. To do this, we provide examples of tasks that have the potential of promoting students' mathematical imagination focusing on the process of bringing mathematical ideas to life through reframing problems, asking questions, connecting and combining, challenging assumptions, changing perspectives and representing ideas (Friedlander, 2016; Leikin & Pitta-Pantazi, 2013).*

## RESEARCH REPORTS

### ENHANCING CREATIVE PROBLEM SOLVING IN AN INTEGRATED VISUAL ART AND GEOMETRY PROGRAM: A PILOT STUDY

Eveline M. Schoevers, MSc<sup>1</sup> & Dr. Evelyn H. Kroesbergen<sup>1</sup>  
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**Abstract.** *This article describes a new pedagogical method, an integrated visual art and geometry program, which has the aim to increase primary school students' creative problem solving and geometrical ability. This paper presents the rationale for integrating visual art and geometry education. Furthermore the MathArt pedagogy and program is described and it is explained how the MathArt program intends to increase students' creative thinking and geometrical ability. Additionally initial results of the pilot study are presented, which investigates the effects of the MathArt program.*

### CAN YOU CHALLENGE ME IN REASONING PROBABILITY, PLEASE?

Efi Papanastasiou<sup>1</sup>, Maria Meletiou-Mavrotheris<sup>2</sup> and Christina Vasou<sup>2</sup>

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**Abstract.** *One of the main issues of teaching talented students is challenging them. The research literature suggests that digital educational games have many potential benefits for mathematics and statistics teaching and learning. One of their foremost qualities is the capacity to motivate, engage, and immerse players. It has been shown that educational games captivate students' attention, contributing to their increased motivation and engagement with mathematics and statistics (Ke, 2008). The current article contributes to the emerging literature on game-enhanced statistics learning by exploring the capabilities of a learning environment that uses programming logic in a game setting. Based on challenging students to create their own games, we attempted to enhance students' (aged between 8 to 13 years old) reasoning about probability by asking them to design a computer game for modeling probabilistic ideas. Students were introduced to the block-based programming language Scratch 2.0, and used it to create their own games. In this article we present the case of a talented 13 year-old boy who expressed many probabilistic ideas while he was designing and playing his game.*

## CREATION OF MATHEMATICAL OBJECTS AS ASPECT OF CREATIVITY IN PRIMARY GRADES

Daniela Assmus & Torsten Fritzlar  
Martin Luther University Halle-Wittenberg

**Abstract.** *In contemporary literature, there is a strong connection between mathematical creativity, problem solving and problem posing. However, the free or purposeful creation of mathematical objects can also be seen as an important aspect of mathematical creativity. In the first theoretical part of the paper, we will further elaborate the concept of mathematical creativity in primary school age. Typical examples of the yet neglected creation of mathematical objects are discussed in the second part.*

## EYE-TRACKING AS A TOOL FOR INVESTIGATING MATHEMATICAL CREATIVITY FROM A PROCESS-VIEW

Maike Schindler & Achim J. Lilienthal  
Örebro University, Sweden

**Abstract.** *Mathematical creativity with its increasing significance in our society and economy is more and more in the focus of mathematics education research. Previous research has focused, among others, on Multiple Solution Tasks (MSTs), where students aim to find many solutions to a given mathematical problem. Whereas most research using MSTs addresses students' products resulting from their creative endeavours (product-view), recent research has asked the question of how students' creative processes look like in which they come up and pursue their creative ideas (process-view). In this paper, we address this question focusing on eye-tracking (ET) as a research method; which has the potential to complement well-established methods in creativity research and reveal insights that may complement the body of research. We give an overview on the opportunities of ET for investigating mathematical creativity. In particular, we discuss the possibilities of different types of ET technology; and how ET using this technology can contribute to studying mathematical creativity.*

## TEACHERS PERCEPTION IN MEETING THE NEEDS OF MATHEMATICALLY GIFTED LEARNERS IN DIVERSE CLASS IN BOTSHABELO HIGH SCHOOLS AT MOTHEO DISTRICT

Mohokare Dimakatso Agnes & Mhlolo Michael Kainose

Central University of Technology – Free State

**Abstract.** *Diversity is one of the country's greatest assets, where teachers are faced with many challenges in their classroom. Many teachers have the willingness to cater for the*

*needs of mathematically gifted learners, but lack the knowledge and skills to be able to do as successfully as they are not adequately prepared enough for effective teaching to meet the needs of gifted learners. The aim of this study was to examine the perception of teachers regarding mathematically gifted learners in diverse class. The study was conducted among 20 high school teacher in Botshabelo. A mixed methods was used and data was collected through questionnaire. The results of the study discovered that teachers' have positive perceptions regarding gifted learners in mathematic based on the characteristics of the gifted and the different provision provided to meet the needs of the gifted and to shed more light on the plight of gifted students in the diverse classrooms.*

## **STRATEGIES USED BY TEACHERS FOR SUPPORTING MATHEMATICALLY GIFTED LEARNERS IN BLOEMFONTEIN HIGH SCHOOLS**

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**Abstract.** *Most gifted learners in mathematics fall through the cracks of inclusive classrooms as are taught by the same methods used to teach other learners. The needs of the gifted learners are often ignored as teachers believe that they can learn on their own without special programs. Teachers' strategy is a key element for educators to provide proper learning instructions to gifted learners. The purpose of this study was to investigate strategies which teachers use when they support mathematically gifted learners in their inclusive classrooms. Nineteen teachers from high schools around Bloemfontein participated in the study. Data was collected through questionnaires.*

## **ANALYSIS OF A GIFTED PRIMARY SCHOOL STUDENT'S ANSWERS TO A PRE-ALGEBRA TEACHING UNIT**

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**Abstract.** *We present a teaching unit aimed to introduce students to algebra and solution of linear equations through geometric pattern problems. We analyse the answers, to the teaching unit, made by a 9-years-old mathematically gifted student in grade 5. We have classified his answers in several styles of strategies of use of graphical information and different types of generalisation. We have also analysed the student's capacity to transfer algebraic knowledge to other algebraic contexts different from geometric pattern problems. Our results show that the student exhibited several characteristics of mathematical giftedness related to his strategies of solution of problems, which are notably different from those strategies that, according to the literature, are used by average students in the same grade.*

## **RE-EXAMINING THE EDUCATIONAL PROVISION FOR MATHEMATICALLY GIFTED STUDENTS ACROSS DIFFERENT POVERTY STRATA IN SOUTH AFRICA**

Michael Kainose Mhlolo

Central University of Technology – Free State South Africa

*Research shows that when provisions are denied to the gifted on the basis that they are elitist, it is the poor who suffer the most because parents from affluent families have options which poorer families cannot afford. In South Africa not much research has been done to understand the extent to which the 'elitist' justification for the dismantling of gifted education has achieved parity and fairness for the poor. This paper analyses the extent to which education provision for gifted students is fairly distributed across the poverty strata. A Qualitative Secondary Analysis (QSA) research design was employed to analyse data in Pan African Mathematics Olympiads (PAMO) documents from 2000 – 2016 together with reports from a Study of the Southern and Eastern African Consortium for Monitoring Educational Quality (SACMEQ). Data were categorised according to the different poverty categories (quintiles) in order to determine whether or not provision for gifted education was fairly distributed. Results show that participants were mainly from quintile five schools with better resources and good teachers while students from poor schools are being taught by teachers who have weak pedagogical knowledge. It can be argued that those who recommended the abolishment of classes for the gifted were in fact penalizing the gifted poor because, as the results confirm, the rich can afford private education. Public school programs for gifted children would allow children who are economically disadvantaged the only opportunities they might have to develop their talents.*

*Key words: inclusive education, gifted students, elitism, quintiles,*

## RESEARCH REPORTS AND PROJECT REPORTS

### HIGHER ORDER THINKING IN MATHEMATICS: A COMPLEX CONSTRUCT

Demetra Pitta-Pantazi and Paraskevi Sophocleous  
Department of Education, University of Cyprus

**Abstract.** *The importance of developing Higher Order Thinking in Mathematics has been an issue of discussion for more than thirty years. However, so far there is no common formulation of higher order thinking in mathematics. During the 1980's the Iowa Department of Education (1989) suggested the Integrated Thinking Model according to which higher order thinking is a combination of Basic, Critical, Creative and Complex Thinking processes. This article aims to address this issue by investigating whether such a formulation can be endorsed for mathematics and whether it can be empirically validated with 691 primary school students using structural equation modeling. The analysis provides support, to a large extent, to the assumptions of this model.*

### WHAT IS THE RELATIONSHIP BETWEEN CRITICAL THINKING AND PROBLEM POSING ABILITY?

Paraskevi Sophocleous and Demetra Pitta-Pantazi  
Department of Education, University of Cyprus

**Abstract.** *The aim of this study is to investigate the relationship between students' mathematical problem-posing ability and their critical thinking in mathematics. The necessity to conduct this study arises because of the limited research findings on this relationship in mathematics education research and the conflicting results from general education research. A group of 127 sixth grade students completed a mathematical critical thinking test and a mathematical problem-posing task. The results showed that students' total score in the mathematical critical thinking test was a statistically significant predictor of their problem-posing fluency and problem-posing flexibility scores. However, students' total score in mathematical critical thinking test did not predict their problem-posing originality score. The results of this study support the position that originality is a more unique characteristic of mathematical creativity.*

### SELF-SELECTED LEARNING COMMUNITIES FOR PROBLEM SOLVING IN ONLINE MATH COURSES

Lynae E. Warren  
State University of New York: Empire State College

**Abstract.** *The purpose of this study is to examine the impact of students' self-selection of their learning community on the depth of learning of cognitively challenging problem solving tasks. Students in multiple sections of an on-line math course were engaged in mathematical problem solving within an instructor-selected (IS) group. Cognitive tasks were selected in order to challenge students so they would seek out the IS group. The intent was for students to develop a relationship with others in IS group in order to engage*

*in dialogic inquiry about how to solve the problems. Using grounded theory and situated learning theories, the researcher found that students who selected their own learning communities, from outside the course, showed a depth of learning that was comparable to that of students who engaged solely in IS groups.*

## **THE ROLE OF THE TEACHER-INVENTOR IN THE DIFFERENTIATED LEARNING ENVIRONMENT**

Maksim Ivanov

University of Tartu, Tartu Annelinna Gymnasium, Estonia

**Abstract.** *The teacher-consultant as the usual role of the teacher in the differentiated learning environment is not attractive or offering any development opportunities for teacher. The more appropriate role for the teacher conducting differentiated instruction, especially of gifted students, is that of the teacher-inventor, who can think up tasks on the spot, is always a few steps ahead, experiments with his students and accepts challenges from them.*

## **MATHEMATICS FROM OLD TIMES AND FOREIGN COUNTRIES – ENCOURAGING MATHEMATICAL CREATIVITY OF PRIMARY STUDENTS THROUGH ETHNOMATHEMATICS**

Torsten Fritzlar, Maria Kötters, Karin Richter  
Martin Luther University Halle-Wittenberg

**Abstract.** *Fostering mathematical creativity in primary school age is a very important but also a challenging task for teachers. Sternberg's Investment Theory of Creativity can give some general hints for this purpose. From the perspective of mathematics education, ethnomathematics could also have some specific potentials in this regard. In this paper, we present a workshop series "Mathematics from old times and foreign countries" combining both approaches.*

## **DEVELOPING MATHEMATICAL CREATIVITY IN PRE-SCHOOL EDUCATION**

Maria Angela Shiakalli, Pre-School Teacher, Cyprus  
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Konstantinos Zacharos, Professor, University of Patras, Greece  
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**Abstract.** *Could creative problem solving be the object of work in pre-school education? This study followed the work of fifteen, four and five year old children and their teacher during a two month process of solving combinatorial problems with a large number of solutions. Findings show that all children responded positively to the problems, were successful in solving them and developed sophisticated strategies during the process.*

## WORKSHOPS

### WHEN THE GAME ENDS, THE MATH BEGINS

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**Abstract.** *Teachers frequently use games to engage students and give them practice in reinforcing mathematical concepts and computation. Too often, however, students do not delve into the mathematics behind the games. In this workshop, participants will begin by playing simple elementary mathematics games, and then use questioning techniques and an open problem-solving/problem-posing heuristic to delve into the mathematics and extend students' reasoning and mathematical creativity.*

### USING PRODUCTIVE STRUGGLE TO DECREASE FRUSTRATION AND INCREASE MOTIVATION

Dr. William Renwick Speer  
University of Nevada Las Vegas

**Abstract.** *The National Council of Teacher's of Mathematics 2014 landmark publication, Principles to action: Ensuring mathematical success for all, connects research with practices that are essential for every student with core principles to build a successful mathematics program at all levels. An important aspect of successful mathematics teaching practices proposed in this book is the examination of unproductive and productive beliefs, obstacles, and key actions that must be understood, acknowledged, and addressed. We can't make our students into seekers if we aren't seekers ourselves. In this research-based, practice-oriented presentation we explore the benefits of creating productive struggle with desirable difficulties to help students shake up naïve or loose thinking and to construct "new" knowledge by transfer of related knowledge to new situations.*

## Wednesday

### RESEARCH REPORTS

#### QUESTIONS ABOUT IDENTIFYING TWICE EXCEPTIONAL STUDENTS IN A TALENT SEARCH PROCESS

Marianne Nolte

University of Hamburg, Germany

***Abstract.** A high potential in a certain (or in several) domain(s) does not necessarily mean a protection against developmental disorders or impairments. In many cases high potentials may mask the special needs of a child and, conversely, special needs may also mask a high potential. Because of the masking effect, even thoroughly executed talent search programs can fail to identify affected students. To learn more about the character and the amount of affected students a questionnaire was given to more than 400 parents whose children participated in a talent search program. The results show that some twice exceptional students can be identified, but the question remains whether identification in group processes is sufficient to give twice exceptional students the possibility to show as high potential as not affected students.*

#### “MATHE MIT PFIFF” – A PROJECT AIMING AT EXTRACURRICULAR ENRICHMENT AT SCHOOL

Ralf Benölken

University of Münster

***Abstract.** In Germany, programs aiming at the support of children who have high mathematical interests and potentials are – irrespective of contests and specific schools – mainly conducted at several universities. Usually, they combine research, the education of student teachers and the support of children. An example is given by the long-term enrichment project “Mathe für kleine Asse” at the University of Münster. By contrast, reports on similar programs conducted at German schools are rare; conceptual exchanges seem to be rather difficult. In this article, the conception of the project “Mathe mit PfiFF” will be presented aiming at a transfer of established cornerstones of “Mathe für kleine Asse” into schools by extracurricular courses.*

**THE LOCAL MATHEMATICAL CONTEST AS AN ASSESSMENT TOOL OF STUDENTS' MATHEMATICAL CAPACITY**

Ingrida Veilande  
Latvian Maritime Academy

**Abstract.** *The paper discusses the mathematical circle at the Correspondence School of Mathematics for primary school students. The circle aims to introduce mathematically interested students to the first steps of problem solving. The composition principles of local mathematical contest are presented. The rationales for evaluating the students' attainments in the contest are discussed. The collected data characterise the levels of students' performance of certain problem-solving skills and suggest guidelines for further running of the mathematical circle.*

**PEDAGOGICAL PRACTICES THAT FOSTER MATHEMATICAL CREATIVITY AT TERTIARY-LEVEL PROOF-BASED COURSES**

<u>Milos Savic</u>	Houssein El Turkey	Gail Tang	Gulden Karakok	Emily Cilli-Turner	David Plaxco	Mohamed Omar
University of Oklahoma	University of New Haven	University of La Verne	University of Northern Colorado	University of Washington Tacoma	University of Oklahoma	Harvey Mudd College

**Abstract.** *Many mathematics education publications highlight the importance of fostering students' mathematical creativity in the tertiary classroom, however few describe explicit instructional methodologies to accomplish this task. Using Sriraman's (2005) five principles for maximizing creativity as a framework, we highlight three case studies for which pedagogical practices occurred to foster creativity. All three proof-based courses used a meta-cognitive tool named the Creativity-in-Progress Rubric (CPR) on Proving. This tool was developed to encourage students to engage in practices that research literature, mathematicians, and students themselves suggest may promote creativity in processes of proving. Centered around the CPR on Proving, we report on the variety of tasks, assignments, and in-class discussions the instructors used in their proof-based courses and verify that these did highlight mathematical creativity using student interview data.*

**DOES INTELLIGENCE AFFECT ALL STUDENTS' MATHEMATICAL CREATIVITY?**

Maria Kattou\* & Constantinos Christou#  
\*Ministry of Education and Culture of Cyprus  
#University of Cyprus

**Abstract.** *The present study aims to investigate whether there is a relationship between intelligence and mathematical creativity and if it does so, in which manner this relation is differentiated between students of different degree of intelligence. Therefore, 476 students attending 4<sup>th</sup>, 5<sup>th</sup> and 6<sup>th</sup> grades of elementary school participated in this study, by completing two tests: one measuring their mathematical creativity and the other measuring their fluid intelligence. Confirmatory factor analysis verified the existence of a moderate relationship between intelligence and mathematical creativity. In particular, intelligence is a factor that might predict an individual's mathematical creativity.*

*Furthermore, correlation analysis indicated that such a relation exists only between students of average intelligence, whereas in students with high or low intelligence, mathematical creative ability and intelligence behaved as independent variables.*

## **SPATIAL ABILITIES AS PREDICTOR TO MATHEMATICS PERFORMANCE OF MATHEMATICS MOTIVATED STUDENTS**

Applebaum, Mark  
Kaye Academic College of Education, Israel

**Abstract.** *In the few years since the Kangaroo Contest arrived in Israel, we discovered that all winners in grades 2-6 had success in tasks oriented on spatial abilities (SA). In this study, we investigate a potential relationship between spatial abilities and mathematics performance (focused on non-standard problems) in mathematically motivated students (MMS) who participated in the Kangaroo Contest. We also checked whether the correlation between scores of SA tasks and the rest non-standard problems (RNSP) in the contest depends on the participants' age. Strong correlation between SA tasks and mathematics performance, as well as well-known malleable spatial abilities can lead us to the necessity of spatial abilities' development in early childhood as predictor of later mathematics achievement. This issue is important for students at all levels and especially for MMS whereas some of them will become later mathematically promising students.*

## **WORKSHOPS**

### **DISCOVERIES WITH DGE: WHICH ONE IS MORE CREATIVE?**

Roza Leikin & Haim Elgrably  
University of Haifa, Israel

**Abstract.** *During the workshop we will share with the participants our experience of "teaching to be creative" implemented with prospective high school teachers. We will present different discoveries that our teachers did and ask the participants of the workshop to decide "What is more creative?" We will introduce our scoring scheme for the evaluation of creativity in the discovery activity. The participants will be asked to discover and evaluate their own discoveries.*

### **ERASMUS+ FUNDING, IDEAS AND NETWORKING**

Gregoris A. Makrides, Vassiliki Savopoulou

Cyprus Mathematical Society, European Association of ERASMUS Coordinators

**Abstract.** *Best Ideas and Innovation cannot be developed without cooperation and co-development. This approach is embedded in EU Funding Programmes as it arises from the requirements of consortia development and project management. This Workshop will present the preparation and management of proposals and projects under the ERASMUS+ Funding Programme and will invite participants in developing ideas for new proposals and form on the spot consortia and leaders for the preparation of proposals to be submitted for the 2018 Call for Proposals, always supporting priorities relating to Mathematics and its Education aspects.*